APPLIED MATHEMATICS 30

Revised Course of Studies

This document contains only the specific outcomes for Applied Mathematics 30. The program rationale and philosophy and general outcomes can be found on pages 1 to 6 of the Applied Mathematics 10–20 program of studies, Interim 1998.

erim 2000

Alberta
LEARNING
Curriculum Standards Branch

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Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples
General Outcome Describe and apply operations on matrices to solve problems, using technology as required. **Specific Outcomes** A6–1. Model and solve problems, including those solved previously, using technology to perform matrix operations of addition, subtraction and scalar multiplication as required. [CN, PS, R, T, V]	1.1 CFL Western Division Standings (23rd August) Against Eastern teams W T L Points BC 1 0 2 2 Calgary 2 0 0 4 Saskatchewan 1 0 1 2 Edmonton 1 0 2 2 Against Western teams W T L Points BC 4 0 0 8 Calgary 3 0 2 6 Saskatchewan 1 0 4 2 Edmonton 1 0 3 2 a) Create a combined table using matrix addition. b) What condition is necessary for there to be a matrix sum $A + B$, given original matrices A and B? 1.2 A store sells items that are tax free, items that have a 7% GST charge on the base price, and items that have both a 7% GST and a 9% PST charge on the base price. A weekend's sales, before tax, is represented in the table: Weekend Sales Summary Sales by Tax Category | Saturday Sales ($) \| Sunday Sales ($) Tax free | 500 | 700 GST only | 1250 | 400 GST and PST | 800 | 700 a) Determine each amount of tax (GST and PST) collected by the store on this weekend, using any method. b) Model and solve the problem, using appropriate matrices derived from the table given. c) How would you change your matrix model for a new GST rate of 5% and a new PST rate of 12%?

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General and Specific Outcomes	Illustrative Examples
	1.3 An arctic expedition outfitter has the following price list for tours of varying lengths and starting points. The prices are quoted in Canadian dollars.

Starting Point	Length of Tour and Cost			
	4 days	7 days	10 days	15 days
Yellowknife	$740	$1145	$1550	$2225
Edmonton	$1640	$2045	$2540	$3125
Toronto	$2240	$2645	$3050	$3725
Vancouver	$1940	$2345	$2750	$3425

a) Research to find a table that compares the Canadian dollar to other currencies.
b) Write a row matrix that represents the cost, in US dollars, of the four tours from a Toronto starting point.
c) Write a matrix, in Japanese yen, for the entire price list.

A6–2. Model and solve consumer and network problems, using technology to perform matrix multiplication, as required. [CN, PS, T, V]

2.1 Singh's Grocery sells several different kinds of breakfast cereal, each at a different price.
Cereal A is 2.65/box.
Cereal B is 3.73/box.
Cereal C is 3.15/box.
Cereal D is 2.99/box.

On Wednesday, they sold the following:
5 boxes of Cereal A
8 boxes of Cereal B
7 boxes of Cereal C
10 boxes of Cereal D.

a) Model and solve this problem, using an appropriate row matrix, and an appropriate column matrix, then use matrix multiplication to determine Wednesday's total revenues.
b) Can the total revenue be calculated, if six boxes of Cereal E (price unknown) were sold? Justify your response.

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General and Specific Outcomes	Illustrative Examples

2.2 Sales of economy cars were 200 in 1998 and fell by 4% in 1999. Sales of midsize cars were 300 in 1998 and rose by 10% in 1999. Sales of luxury cars were 40 in 1998 and rose by 30% in 1999.

a) Represent the 1998 and 1999 sales by appropriate column matrices.
b) Represent the changes from 1998 to 1999 by an appropriate matrix multiplication.

2.3 Soccer's administrators have been experimenting with using league standings to discourage tie games, especially those in which no goals are scored. The traditional scheme of 2 points for a win and 1 point for any tie has been replaced by a new scheme that awards 3 points for a win and 1 point for all ties. Two other proposed schemes are 3 points for a win, 1 point for ties that have goals scored and 0 points for ties with no goals; and a scheme with 5 points for a win, 3 points for a tie with goals scored and 0 points for a tie with no goals. In a local soccer league the top four team records after 42 games are:

	Wins	Ties with Goals	Ties with No Goals	Losses
Tigers	30	2	8	2
Irish	24	9	2	7
Colts	25	7	0	10
Jets	26	1	10	5

a) Determine the points for each team, using the traditional scheme.
b) Model this solution, using matrix multiplication.
c) Using matrices, model and determine the points for each team, using the new scheme. Repeat for the other two proposed schemes.
d) Which of the alternative scoring systems can make the Irish second in the standings?
e) Which of the alternative scoring systems can make the Colts second in the standings?
f) Which of the alternative scoring systems can make the Jets second in the standings?
g) Design a system that would drop the Tigers out of first place.

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General and Specific Outcomes	Illustrative Examples

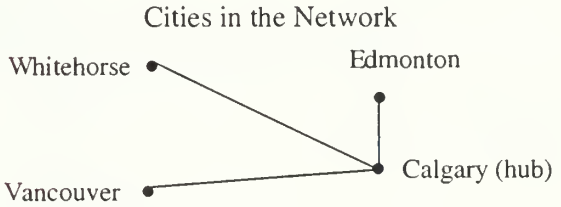
2.4 A washing powder is sold in 6 L and 10 L packages. Market research shows that 40% of the users of the 6 L size switch to the 10 L size for their next purchase, and 20% of the users of the 10 L size switch to the 6 L size for their next purchase.

a) If the original market share was 60% for 6 L and 40% for 10 L, what is the market share for each size in the next round of purchases?

b) What is the market share for each size for the third round of purchases?

2.5 Air freight is transported among several cities in Western Canada, using Calgary as the hub. This means that all air freight either originates in Calgary or is transported through Calgary.

a) Set up a network matrix A for the following cities, using 1 to represent a direct freight transport between two cities and 0 to indicate there is no direct freight transport between two cities.



b) The matrix A^2 can represent the network matrix for those routes with exactly one stop. Evaluate the matrix A^2.

c) Explain why there is one element equal to 3, nine elements equal to 1 and six elements equal to 0 in the matrix A^2.

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General and Specific Outcomes	Illustrative Examples
General Outcome Design or use a spreadsheet to make and justify financial decisions. **Specific Outcomes** A8–1. Design a financial spreadsheet template to allow users to input their own variables. [C, PS, T]	1.1 For the following invoice, develop a spreadsheet that calculates the totals and that requires the operator to input a minimum number of entries.

ACME AUTO PARTS

Customer Inquiries

Item No.	Auto Parts	Quantity	Unit Price	Total	Labour	
1	Brake Pads	1	26.34	26.34	O/H Front Brakes (0.9 hrs. @ 53.00/hr.)	47.70
2	Wheel Seals	2	5.25	10.50	Machined and Replaced Rotor (flat rate)	10.00
3	Rotor	1	30.16	30.16		
					Total Labour	57.70
					Total Parts	67.00
		Total Parts		67.00	PST on Parts (8%)	5.36
					GST (7%)	8.73
					TOTAL	138.79

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General and Specific Outcomes	Illustrative Examples
A8–2. Analyze the costs and benefits of renting or buying an increasing asset; e.g., home, under different circumstances. [C, CN, PS, T]	2.1 The Wong family is faced with a move and has the choice of buying a home for $145 000 with a $25 000 down payment, or renting a similar house for $975 per month. Four options are available. 1. Buy the house with a 20-year mortgage and continue investing at the same rate after the mortgage is paid. 2. Buy the house with a 30-year mortgage. 3. Rent a house and invest the $25 000. 4. Rent a house and invest both the $25 000 and the difference each month between the rent and the mortgage payment. Set up analysis spreadsheets that include the following inputs: a) mortgage interest rate, taking 8.5% as a starting value b) taxation rate, taking 1.5% of market value as a starting value c) annual rent increase, taking 5% per annum as a starting value d) annual increase in house value, taking 4% per annum as a starting value e) investment return, taking 7.0% as a starting value. Try different scenarios, varying from 1 year to 30 years. Summarize circumstances in which buying makes sense, and summarize circumstances when renting makes sense.
A8–3. Analyze the costs and benefits of leasing or buying a decreasing asset; e.g., vehicle, computer, under different circumstances. [C, CN, PS, T]	3.1 A car lease runs for 36 months at $305 per month, with a down payment of $1105, a lease-end value of $7105 and an interest rate of 11.6%. Maintenance is the purchaser's responsibility. Set up a spreadsheet to include the monthly values of the opening balance, interest paid, lease payment and closing balance. Use the spreadsheet to answer the following questions. a) What part of the $305 is used to pay the interest on the $7105? b) What total price is being charged for the car? c) What is the change in the monthly lease payment, if the lease-end value is reduced to $5700? d) What is the monthly payment for a straight purchase over 36 months with a 20% down payment? e) What is the annual percentage depreciation rate assumed with the $7105 lease-end value?

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General and Specific Outcomes	Illustrative Examples
A8–4. Analyze an investment portfolio applying such concepts as interest rate, rate of return and total return. [C, CN, PS, T]	4.1 The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years. a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates: • 4% per annum, compounded annually • 8% per annum, compounded annually • 24% per annum, compounded annually. b) What general conclusion can be drawn as to the accuracy of the rule of 72 calculations? 4.2 Annette makes several investments in her RRSP. She invests $5000 in a GIC paying 6%, compounded semi-annually, when she is 25 years old. At the age of 30, she places $12 000 in a GIC paying 8 ½ %, compounded annually. On her 35th birthday, she invests $8000 in a GIC paying 7%, compound semi-annually. a) What will her GICs be worth when she is 65, if she keeps them invested at the rates given until her 65th birthday? b) What is the average annual rate of return over the last 30 years of the investment? 4.3 An investment of $10 000 is held from December 31, 1995 to December 31, 1998. It contains 75% Canadian stocks that track the Toronto Stock Exchange (TSE) 300 composite index, and 25% invested in guaranteed investment certificates (GICs) that pay 6.5% compounded annually. On December 31, 1995 the TSE 300 composite index was at 4714. On December 31, 1998, the TSE 300 composite index was at 6486. a) What was the value of the total portfolio on December 31, 1998? b) What average annual rate of return does this represent?

Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

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General and Specific Outcomes	Illustrative Examples
General Outcome Generate and analyze cyclic, recursive and fractal patterns. **Specific Outcomes**	
A7–1. Collect sinusoidal data; graph the data using technology, and represent the data with a best fit equation of the form: • $y = a \sin(bx + c) + d$. [C, CN, PS, T, V]	1.1 Research the sunrise time, each day, for a period of one year, and graph it. From your graph, determine the time of sunrise for March 12. 1.2 Collect data from real-world situations, such as: a) hours of daylight b) low tide and high tide c) average low and average high temperatures on different dates of the year. Plot the data, and determine an approximate equation for the data in the form of: $y = a \sin(bx + c) + d$.
A7–2. Use best fit sinusoidal equations, and their associated graphs, to make predictions (interpolation, extrapolation). [C, CN, PS, T]	2.1 Sketch a graph, and build an equation to represent the following situation. From Environment Canada, find the average daily maximum temperatures on the 1st and 15th day of each month for one year, in a city of your choice. Use a sinusoidal equation to predict: a) the highest daily average maximum temperature b) the lowest daily average minimum temperature c) when the highest temperature will occur d) when the lowest temperature will occur.

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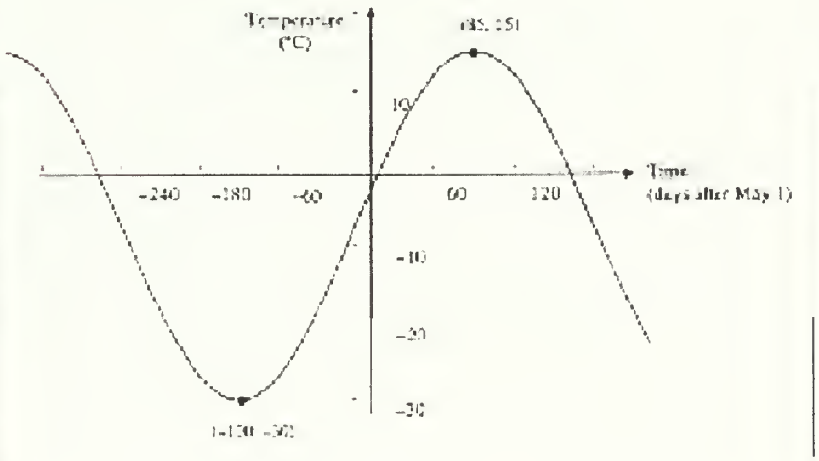
General and Specific Outcomes	Illustrative Examples
A7–3. Describe periodic events, including those represented by sinusoidal curves, using the terms amplitude, period, maximum and minimum values, vertical and horizontal shift. [C, V]	3.1 A temperature–time graph was drawn for a northern Saskatchewan town. The variable plotted on the horizontal axis is the calendar date, with April 1 as zero and the unit being days. The variable plotted on the vertical axis is the temperature in degrees Celsius. The graph is drawn below. Find the: a) amplitude b) period c) maximum and minimum values d) vertical shift e) date for the maximum temperature f) date for the minimum temperature. 

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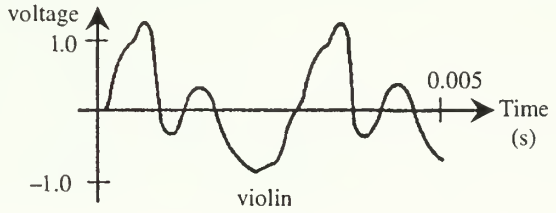
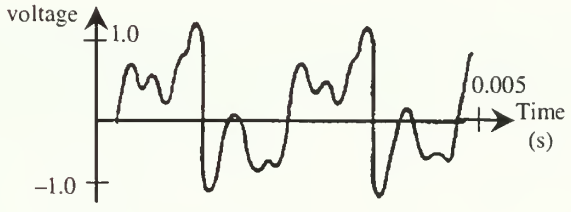
General and Specific Outcomes	Illustrative Examples

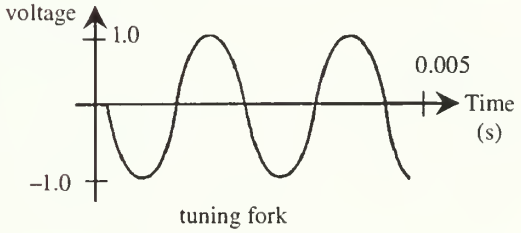
3.2 The following are graphs showing the patterns produced on an oscilloscope when four different musical instruments are played.



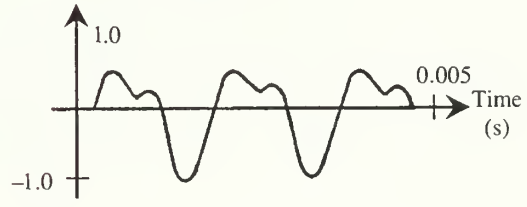
violin



clarinet



tuning fork



organ pipe

From *Fundmentals of Physics* by Martindale et al. Reprinted by permission of ITP Nelson Canada.

For each instrument:
a) estimate the amplitude
b) estimate the period
c) sketch an approximate form for the graph, if the instrument is played louder
d) sketch an approximate form for the graph, if the instrument is used to play a higher note.

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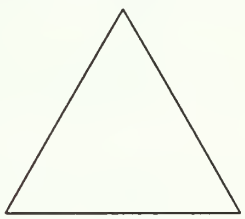
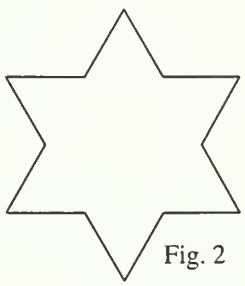
General and Specific Outcomes	Illustrative Examples
A7–4. Use technology to generate and graph sequences that model real-life phenomena. [PS, T, V]	4.1 A mild infection spreads through a school. On any given day 10% of the uninfected population becomes infected, and 20% of the infected population is cured. Once cured, there is zero chance of further infection. On the first day of school, 100 students are infected, 900 students are uninfected, and zero students are cured. a) Track the epidemic for 30 days. b) Graph numbers of infected, uninfected and cured as a function of time for 30 days.
A7–5. Use technology to construct a fractal pattern by repeatedly applying a procedure to a geometric figure. [CN, R, V, T]	5.1 The following example is the Koch snowflake curve. Construct an equilateral triangle (Fig. 1). Trisect each side, construct an equilateral triangle on each middle third, and delete the middle third (Fig. 2).  Fig. 1  Fig. 2 For each segment in Fig. 2, repeat the above. 5.2 Construct your own fractal pattern.
A7–6. Use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns. [CN, R, T, V]	6.1 For illustrative example 5.1, predict the perimeter of the fifth pattern.

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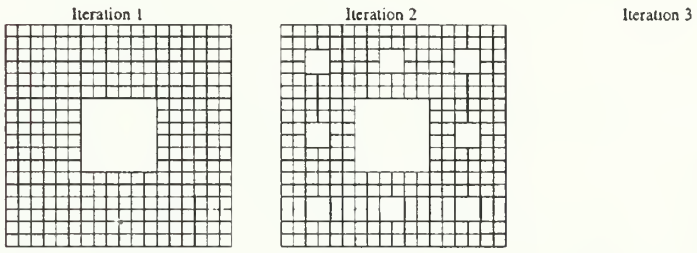
General and Specific Outcomes	Illustrative Examples
	6.2 Fractal Carpet A fractal can be generated by a pattern of iteration. This fractal design is called the Sierpinski carpet after the mathematician who invented it in 1916. The general rule is to start with a square and take a square out. Look at the first iteration and describe the rule that was used to determine the size of the square that was removed. Now compare the first two iterations and describe the rule that was used to construct the second from the first. Apply the rule you have stated to construct the third iteration in the space provided.  Now examine the third iteration you have constructed, and record the length of the side of the new squares you drew. Compare this length to the lengths of the sides of the previous squares. Write the lengths of the sides of all the squares in descending order. If you construct the fourth iteration, what will the lengths of the sides of the squares need to be? Now look at the first iteration again. What is the area of the square that was removed? What is the area of each individual square that was removed in the next two iterations? Write these areas in descending order. What is the area of each individual square to be removed in the fourth iteration? Challenge: Find the perimeter of all the squares in the third iteration. Find the area of the figure that remains once all the squares are removed in the third iteration.

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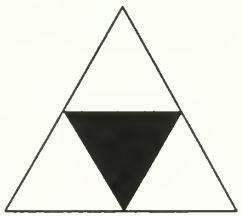
General and Specific Outcomes	Illustrative Examples
	6.3 The Sierpinski triangle can be created by using dilations and isometries. You may begin with an arbitrary triangle. An equilateral triangle is used for the procedures described below. a) Draw an equilateral triangle. b) Reduce the triangle by a factor of ½. Make three copies of the reduced triangle. c) Place the three reduced similar triangles on the original, one at each vertex. d) Eliminate the remaining portion of the original triangle by blackening it.  Your work should result in the figure shown here. Answer the following questions: a) Let the area of the original triangle be 1 area unit. What area remains? What area has been removed? b) Let the side of the original triangle be 1 length unit. What is the perimeter of the figure with the dark region removed? Repeat steps a) through d) of the original procedure for each of the triangular regions remaining in the figure shown. Sketch the result of your work. Answer the following questions: a) What is the area of the remaining triangular region? b) What is the perimeter of the new "holey" triangular region? c) What would the next iteration of the procedure look like? Make a sketch. d) Write an expression for the area of the Sierpinski triangle after carrying out the procedure n times. e) Write an expression for the perimeter of the Sierpinski triangle after carrying out the procedure n times. f) How would your expressions differ, if you began with a triangle other than an equilateral triangle? Excerpted and adapted with permission from *Geometry from Multiple Perspectives* (*Curriculum and Evaluation Standards for School Mathematics: Addenda Series*, Grades 9–12), copyright 1991 by the National Council of Teachers of Mathematics. All rights reserved.

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General and Specific Outcomes	Illustrative Examples
	6.4 Construct a cylinder with the dimensions: $r = 10$ cm, $h = 20$ cm. A second figure is constructed by halving the previous radius and height. A third is constructed by halving the second and so on. a) Predict the surface area and the volume of the sixth pattern. b) Write an expression for the surface area after carrying out the procedure n times. c) Write an expression for the volume after carrying out the procedure n times.

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

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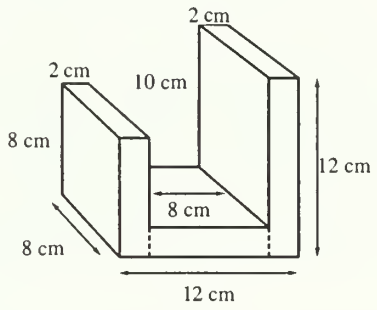
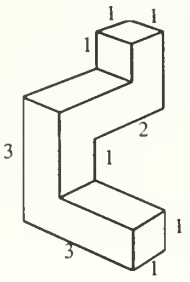
General and Specific Outcomes	Illustrative Examples
General Outcome Analyze objects, shapes and processes to solve cost and design problems. **Specific Outcomes** A9–1. Use dimensions and unit prices to solve problems involving perimeter, area and volume. [E, PS, V]	1.1 Determine the volume of the plastic book end shown below.  If the book end is constructed using an injection mold, find the materials cost, if the plastic ingredients cost 6¢ per cubic centimetre. 1.2 The following diagram shows a structure for enclosing a cable system. All the angles are right angles and the lengths are in decimetres. A special latex coating is applied to all outside surfaces of the structure. What is the cost of the latex coating, if it costs 28¢ per cm^2? 

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
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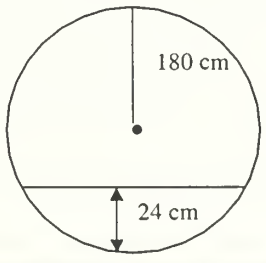
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples
A9–2. Solve problems involving estimation and costing for objects, shapes or processes when a design is given. [C, E, PS]	2.1 A swimming pool is 50 m by 21 m. The deep end is 4.0 m deep and extends out 12 m. The shallow end is 1.2 m deep and extends out 12 m. There is a uniform slope connecting the deep and shallow ends. a) Draw scale diagrams showing the top view and the side view of the pool. b) Calculate the cost of filling the pool with water at \$2.00/m^3. c) Waterproofing of the underwater surfaces costs \$17/m^2. Determine the cost of waterproofing. 2.2 To satisfy the building code, an auditorium has to have 1200 m^2 of washroom space. In a washroom for males, the average space needed is 1.9 m^2 per user and the average usage time is 97 s. In a washroom for females, the average space needed is 2.4 m^2 per user and the average usage time is 145 s. Determine the required washroom space: a) on the basis of equal areas for males and females b) on the basis of equal users per hour for males and females.
A9–3. Design an object, shape, layout or process within a specified budget. [PS, R, V]	3.1 Tin plate for making cylindrical cans comes in sheets that are 240 cm by 160 cm and costs \$3.20 per sheet. Cans are 6 cm in diameter and 11 cm high, and they have 3 seals each. Seals cost 0.8¢ each to make. One sheet of tin plate is used for making pieces for ends, and two sheets are used for making pieces for sides. a) How many ends and how many sides can be made from the three sheets of tin plate? b) How many cans can be made from the three sheets, and what is the cost per can? c) Is there another way of making more cans from the three sheets, or the same number of cans from less tin plate? d) How much money is saved doing it the second way? 3.2 To produce a voters' list for a city riding, a sum of \$1.70 per voter is allocated. Four methods of enumerating are possible: Method / Cost per Voter / Probability of Return Hand deliver enumeration form, mail return — \$0.91 — 0.700 Mail form both ways — \$1.07 — 0.740 Telephone until voter reached — \$2.21 — 0.920 Enumerator visits until voter at home — \$5.26 — 0.995 For a riding of 40 000 potential voters, find the maximum number of voters who can be enumerated within the budget and the minimum budget needed to be sure of enumerating 98% of the potential voters.

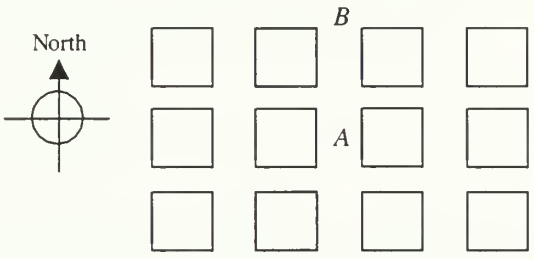
Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples
	3.3 A window cleaner has been asked by the owner of a large office tower to submit a quotation for cleaning the windows of the building. The window cleaner has the following information: a) there are 24 floors b) there are 14 windows per side on each floor c) there are 4 sides to the building. From experience, the window cleaner knows that the transfer time between windows on the same floor and same side of the building is 60 seconds. The transfer time between sides of the building is 120 seconds and between floors is 30 seconds. The time to clean one window is 120 seconds. The window cleaner has a base charge of $120. The maximum period of time he works at one stretch is 3 hours, then he takes a 30-minute rest. In addition to his rate of $25/hour, he wants to make 25% profit from the job for reinvestment in his business. What would be the best quote?
A9–4. Use models to estimate the solutions to complex measurement problems. [E, V]	4.1 Obtain a Mercator projection map and a polar projection map of Canada. Use a transparent overlay grid to estimate the area of the Yukon by counting squares. a) Which type of map gives the most accurate estimate of the area? Explain the sources of error when the estimates are compared to the exact area given in a reference source. b) Estimate the area by splitting it into rectangles and triangles, and compare this method to the previous one you used. Which method is most accurate? Which type of map gives the most reliable estimate for the area of the Yukon? Where are the main sources of error in the estimate? 4.2 A water tank is a sphere of diameter 3.6 m. a) Estimate the volume of water in the tank, if the depth of water is 24 cm. b) Describe the estimation process you used. 

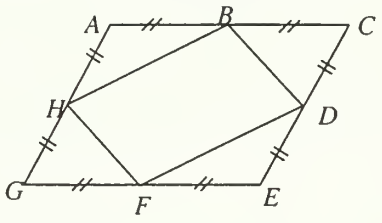
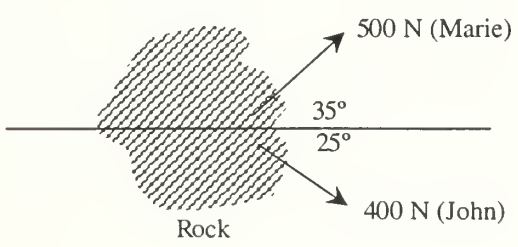
Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples
General Outcome Solve problems involving polygons and vectors, including both 3-D and 2-D applications. **Specific Outcomes**	
A6–3. Use appropriate terminology to describe: • vectors; i.e., magnitude, direction • scalar quantities; i.e., magnitude. [C, CN]	3.1  A car at location A is travelling toward location B. The speed is constant at 50 km/h. a) Describe the speed in terms of a scalar. b) Describe the velocity in terms of a vector.
A6–4. Assign meaning to the multiplication of a vector by a scalar. [CN]	4.1 The vector $\vec{a}$ represents a velocity of 40 km/h east. Make a scale drawing of each of the following vectors: a) $3\vec{a}$ b) $7\vec{a}$ c) $-3\vec{a}$ d) $1.6\vec{a} + 4\vec{a}$. What velocities do each of these vectors represent?

Strand: Space and Shape (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples
A6–5. Determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods. {CN, R, T, V]	5.1  Using the above diagram of a rhombus *ACEG*, determine the vector addition of each of the following: a) $\vec{AH} + \vec{HG}$ b) $\vec{GF} + \vec{BC}$ c) $\vec{GF} + \vec{CB}$ d) $\vec{FD} + \vec{DE}$. 5.2 A ski jumper encounters a horizontal friction of 85 N backward, a vertical weight of 750 N downward and an air resistance of 340 N upward. Draw the vector addition of these forces, and use the drawing to find the magnitude and direction of the resultant force. 5.3 A boat is travelling across a river with a forward velocity of 14 m/s, and there is a current of 3 m/s down the river. How fast is the boat travelling? 5.4 John and Marie are using two ropes held horizontally to try to pull a rock on a horizontal plane. a) Draw a vector diagram to estimate the magnitude and direction of the resultant applied force. b) Verify the estimate by a calculation. c) If the rock does not move, what is the magnitude and direction of the force of friction? Top View 

Strand: Space and Shape (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples
A6–6. Model and solve problems in 2-D and simple 3-D, using vector diagrams and technology. [CN, PS, T, V]	6.1 Model, by drawing a diagram, Jack's jogging route, if he jogs north at 15 km/h for 30 minutes and then turns east and jogs at 12 km/h for 20 minutes. How far has he jogged in total? How far is he from his starting point? In what direction does he need to go to return to the start by the shortest path? 6.2 An aircraft flying horizontally on a heading of 285° is pushed by a wind from 195°. Angles are measured clockwise from north. The indicated air speed of the aircraft is 300 km/h. The wind is constant at 90 km/h. After 1 hour and 15 minutes of flight, what will be the aircraft's change in location? 6.3 An aircraft flies at 80 m/s due north and climbs at an angle of 20° with the horizontal. A horizontal wind blows from west to east at 30 m/s. a) What is the magnitude of the velocity of the aircraft? b) What is the angle that the aircraft's track makes with the horizontal?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples
General Outcome Use normal and binomial probability distributions to solve problems involving uncertainty. **Specific Outcomes**	
C6–1. Find the population standard deviation of a data set or a probability distribution, using technology. [CN, E, T, V]	1.1 Measure the height of each student in a class, and calculate the mean and standard deviation. 1.2 A company uses an automated packaging device to produce 50-gram bags of Karmel Korn. The machine needs frequent checking to see if it is actually putting 50 g in each bag. The following are the masses, in grams, of thirty bags of Karmel Korn. 54 50 47 50 51 50 53 50 47 51 50 51 52 49 46 52 50 49 52 48 48 53 49 49 51 48 49 52 49 50 a) Calculate the mean and standard deviation of this data. b) What problems will be encountered, if the standard deviation gets too high? Dottori et al., *Foundations of Mathematics 11*, p. 391. Adapted with permission.
C6–2. Use z-scores and the normal distribution to solve problems. [PS, R, T, V]	2.1 The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 1.5 mL. a) Calculate the z-score for a can with a volume of 355 mL. b) What percentage of production will consist of cans having content volumes between 350 mL and 355 mL? c) What percentage of production will consist of cans having content volumes less than 355 mL? d) If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected in a run of 50 000?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples
	2.2 For entry into the Canadian Armed Forces, the standards for height were previously set at 158 cm to 194 cm for males, and 152 cm to 184 cm for females. Use the concept of z-score to test if these two height standards are equivalent. Assume that heights for males and for females are normally distributed with means of 176 cm and 163 cm and standard deviations of 8 cm and 7 cm respectively. 2.3 A sample of 122 people gives a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find: a) the expected number of people with temperatures above 37.0°C b) the expected number of people with temperatures below 36.0°C. Also, estimate the range of temperatures contained within the sample. 2.4 In the general population, the IQ scores of individuals is normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested: a) What proportion of this group is expected to have IQs between 100 and 120? b) What is the probability that an individual in the group has an IQ greater than 120?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

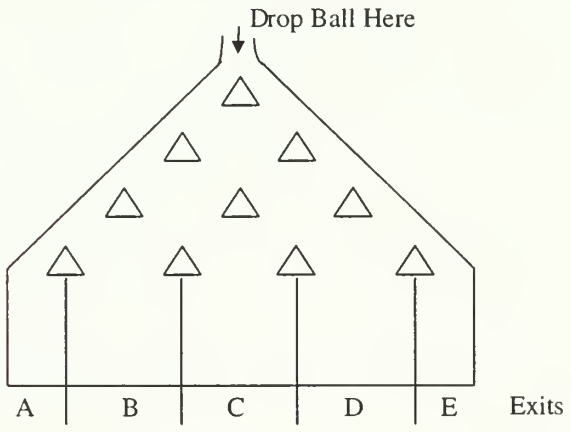
General and Specific Outcomes	Illustrative Examples
C6–3. Use the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples. [CN, E, PS, T]	3.1 Based on past experience, a car salesperson will complete sales to 10% of the customers that enter the showroom. If the salesperson has 200 customers in the next month, establish a symmetric 95% confidence interval for the number of completed sales for the month. 3.2 A true/false test has 40 questions. Assume all questions are answered by guessing. Use a normal distribution to estimate the probability of guessing 25 or more questions correctly. 3.3 From a sample of 250, pollsters estimate that the number of decided voters in favour of a particular provincial law is 64%, and the number opposed is 36%. Discuss how the percentage margin of error changes as the sample size changes.

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples
General Outcome Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations. **Specific Outcomes**	
C6–4. Solve pathway problems, interpreting and applying any constraints. [PS, R]	4.1 Given the following "pinball" situation, what is the probability of the ball reaching each of the exits?  What assumptions are made in the solution?
C6–5. Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]	5.1 Joe has three different shirts, two different pairs of pants and five different pairs of shoes. List all possible outfits in such a way as to ensure that all have been counted and none have been counted twice. How many possible outfits are there? Use the fundamental counting principle to determine the number of outfits there should be. Do your answers match? 5.2 An airline pilot reported that in seven days she spent one day in Winnipeg, one day in Regina, two days in Edmonton and three days in Yellowknife. How many different itineraries are possible? What difference would it make if the first day and the last day had to be spent in Yellowknife?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples
General Outcome Model the probability of a compound event, and solve problems based on the combining of simpler probabilities. **Specific Outcomes**	
C6–6. Construct a sample space for two or three events. [PS, R, V]	6.1 List the sample space for rolling a 6-sided die and flipping a coin. 6.2 Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15 inclusive. A train is scheduled to arrive between 07:11 and 07:17 inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the point (6, 14). Times are expressed in whole minutes. a) How many points are there in this sample space? b) How many points have the bus and the train arriving at the same time? c) How many points have the bus arriving after the train? d) What is the probability of the bus arriving after the train?
C6–7. Classify events as independent or dependent. [C]	7.1 Classify the following events as independent or dependent: a) tossing a head in a coin toss and rolling a 6 on a die b) drawing an ace for the first card and another ace for the second, if the experiment is carried out without replacement c) drawing a king for the first card and a queen for the second, if the experiment is carried out with replacement. 7.2 Sixty per cent of young drivers take driver training, and 25% of young drivers have an accident in their first year of driving. Statistics show that 10% of those who do take driver training have an accident in their first year. Are taking driver training and having an accident in the first year independent events? Support your answer with the appropriate mathematical calculations.
C6–8. Solve problems, using the probabilities of mutually exclusive and complementary events. [CN, PS, R]	8.1 If the probability of winning a game is $\frac{1}{31}$, what is the probability of losing the game?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples
	8.2 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot?

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